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LETTER TO THE EDITOR

# A non-singular, local $^1S_0$ neutron-proton potential

M W KERMODE

Department of Applied Mathematics, University of Liverpool, PO Box 147, Liverpool L69 3BX, UK

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**Abstract.** A non-singular, local $^1S_0$ n–p potential is introduced. This potential gives phase shifts for n–p scattering which agree, within experimental error, with the experimental phase shifts.

The purpose of this letter is to introduce a local potential which gives a good fit to the $^1S_0$ n–p phase shifts ($\delta$) of MacGregor *et al* (1969). This is a report of the first stage of work in which it is hoped that similar potentials for the other angular momentum and isospin N–N states can be obtained. This work is an attempt to extend that of Hamada and Johnston (1962) and Reid (1968) to include potentials with shapes not unlike those found by Sprung and Srivastava (1969) and at the same time to obtain a better fit to the experimental data. The resulting potentials should then be very useful for nuclear calculations, for example, the three-body problem (Hennell and Delves 1972).

The n–p potential $V$ is a very complicated quantity and it will probably be a long time before it can be determined. In principle, once $V$ is known the Schrödinger equation can be solved to give phase shifts at all energies and from these phase shifts one can, again in principle, solve the inverse scattering problem and construct an equivalent local potential $V(r)$, that is, one which gives the same phase shifts as $V$. Here $r$ is the distance between the neutron and the proton.

In the present work it is taken for granted that $V(r)$ exists. An approximation to $V(r)$ is constructed using the following arguments. It is known that the n–p potential $V$, and hence $V(r)$, has as its asymptotic form, the well known OPE potential, that is,

$$V(r) \sim -14{\cdot}947\,\mathrm{e}^{-\mu r}/r, \qquad \mu = 0{\cdot}7\,\mathrm{fm}^{-1}.$$

Suppose that $f(r) = rV(r)$ is a sufficiently smooth function. Then under the transformation $x = \mathrm{e}^{-\mu r}(f(r) \to g(x))$, $g(x)$ can be written in terms of Legendre polynomials, that is,

$$g(x) = \sum_{n=0}^{\infty} a_n \mathrm{P}_n(x) = \sum_{n=1}^{\infty} b_n x^n.$$

Hence

$$f(r) = \sum_{n=1}^{\infty} b_n\,\mathrm{e}^{-n\mu r}.$$

Furthermore, since the phase shifts are not known beyond a laboratory energy of 460 MeV, the potential $V(r)$ cannot be well determined in the vicinity of $r = 0$. Therefore, we take $V(0) = 0$.

Table 1.

| $n$ | $V(n)$ (MeV fm) | $V(n+1)$ (MeV fm) | $V(n+2)$ (MeV fm) |
|---|---|---|---|
| 1 | 14·947 | −190·252 | 1634·987 |
| 4 | −1041·816 | 2798·761 | −16629·85 |
| 7 | 2412·788 | −24141·64 | 159461·9 |
| 10 | 145437·0 | 146515·4 | −2937508 |

Table 2.

| $E_{lab}$ (MeV) | $\delta_{exp}$ (deg) | $\delta_{pot}$ (deg) | $\Delta\delta_{exp}$ (deg) | $\lvert\delta_{exp}-\delta_{pot}\rvert$ (deg) |
|---|---|---|---|---|
| 1 | 62·43 | 62·43 | 0·01 | 0·00 |
| 2 | 65·03 | 65·05 | 0·03 | 0·02 |
| 3 | 65·35 | 65·40 | 0·06 | 0·05 |
| 4 | 65·06 | 65·12 | 0·08 | 0·06 |
| 5 | 64·53 | 64·61 | 0·11 | 0·08 |
| 6 | 63·91 | 63·99 | 0·14 | 0·08 |
| 8 | 63·57 | 62·65 | 0·20 | † |
| 10 | 61·23 | 61·31 | 0·26 | 0·08 |
| 12 | 59·95 | 59·99 | 0·32 | 0·04 |
| 14 | 58·73 | 58·73 | 0·37 | 0·00 |
| 16 | 57·56 | 57·52 | 0·42 | 0·04 |
| 18 | 56·46 | 56·36 | 0·47 | 0·10 |
| 20 | 55·41 | 55·25 | 0·52 | 0·16 |
| 25 | 52·96 | 52·66 | 0·62 | 0·30 |
| 30 | 50·73 | 50·29 | 0·71 | 0·44 |
| 40 | 46·72 | 46·07 | 0·86 | 0·65 |
| 50 | 43·16 | 42·39 | 0·98 | 0·77 |
| 60 | 39·90 | 39·11 | 1·10 | 0·79 |
| 70 | 36·89 | 36·15 | 1·21 | 0·74 |
| 80 | 34·08 | 33·44 | 1·33 | 0·64 |
| 90 | 31·45 | 30·95 | 1·44 | 0·50 |
| 100 | 28·97 | 28·64 | 1·55 | 0·33 |
| 120 | 24·41 | 24·45 | 1·74 | 0·04 |
| 140 | 20·31 | 20·73 | 1·88 | 0·42 |
| 160 | 16·62 | 17·38 | 1·97 | 0·76 |
| 180 | 13·27 | 14·32 | 2·02 | 1·05 |
| 200 | 10·22 | 11·50 | 2·04 | 1·28 |
| 220 | 7·45 | 8·87 | 2·02 | 1·42 |
| 240 | 4·92 | 6·42 | 2·00 | 1·50 |
| 260 | 2·61 | 4·11 | 1·97 | 1·50 |
| 280 | 0·49 | 1·93 | 1·96 | 1·44 |
| 300 | −1·46 | −0·15 | 1·98 | 1·31 |
| 320 | −3·25 | −2·12 | 2·04 | 1·13 |
| 340 | −4·90 | −4·01 | 2·16 | 0·89 |
| 360 | −6·41 | −5·82 | 2·32 | 0·59 |
| 380 | −7·81 | −7·55 | 2·53 | 0·26 |
| 400 | −9·10 | −9·23 | 2·78 | 0·13 |
| 420 | −10·29 | −10·84 | 3·07 | 0·55 |
| 440 | −11·39 | −12·40 | 3·39 | 1·01 |
| 460 | −12·41 | −13·91 | 3·73 | 1·50 |

† Perhaps the value of $\delta_{exp}$ at 8 MeV should be 62·57°?

The potential $V(r)$ was approximated by the potential

$$W(r) = -\sum_{n=1}^{14} V(n)\frac{e^{-n\mu r}}{r},$$

where $V(1) = 14{\cdot}947$ MeV fm and $V(13)$ and $V(14)$ were chosen to make $W(0) = 0$. The values of $V(n)$ listed in table 1 give the phase shifts $\delta_{\text{pot}}$ in column 3 of table 2. Each of these phase shifts is in very good agreement with the corresponding experimental value $\delta_{\text{exp}} \pm \Delta\delta$ (except at 8 MeV where it would appear that $\delta_{\text{exp}}$ should be 62·57°).

The potential $W(r)$ is zero at the origin, has a maximum value of about 11440 MeV at 0·09 fm, is zero at 0·57 fm, has a minimum value of about $-139$ MeV at 0·72 fm and tends to zero as $r$ tends to infinity.

It is interesting to note that if it were not for the sign of $V(10)$ the signs of the $V(n)$ would alternate. It is even more interesting to note that for the expansion in terms of Legendre polynomials, that is

$$W(r) = -r^{-1}\sum_{n=0}^{14} U(n)\mathrm{P}_n(e^{-\mu r}),$$

the signs of the $U(n)$ do alternate. Truncated values of these coefficients are given in table 3.

**Table 3.**

| $n$ | $U(n)$ (MeV fm) | $U(n+1)$ (MeV fm) | $U(n+2)$ (MeV fm) |
|---|---|---|---|
| 0 | $-3{\cdot}36\times10^{5}$ | $+9{\cdot}38\times10^{5}$ | $-1{\cdot}36\times10^{6}$ |
| 3 | $+1{\cdot}53\times10^{6}$ | $-1{\cdot}47\times10^{6}$ | $+1{\cdot}25\times10^{6}$ |
| 6 | $-9{\cdot}40\times10^{5}$ | $+6{\cdot}36\times10^{5}$ | $-3{\cdot}83\times10^{5}$ |
| 9 | $+2{\cdot}06\times10^{5}$ | $-9{\cdot}70\times10^{4}$ | $+3{\cdot}93\times10^{4}$ |
| 12 | $-1{\cdot}35\times10^{4}$ | $+3{\cdot}38\times10^{3}$ | $-7{\cdot}21\times10^{2}$ |

Full details of the method used to determine the $V(n)$ will be given when the $^1S_0$ p–p phase shifts have been fitted: the potential given in this letter does not provide a good fit to the p–p data. The p–p case is slightly more complicated since the Coulomb and vacuum polarization potentials must be taken into account.

I wish to thank Professor L M Delves for suggesting that I look for local potentials to fit the nucleon–nucleon scattering data.

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